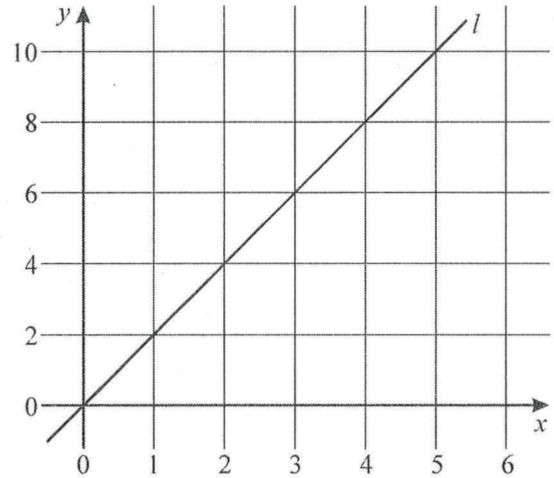
IB Math Studies SL 11 Ch13 Coordinate Geometry And Geometry Review **Name** ___________

1. The following diagram shows a straight line l.



(a) Find the equation of the line l.

(b) The line n is parallel to l and passes through the point (0, 8). Write down the equation of the line n.

(c) The line n crosses the horizontal axis at the point P. Find the coordinates of P.

(Total 4 marks)

2. **A is the point (2, 3), and B is the point (4, 9).**

(a) Find the gradient of the line segment [AB].

(b) Find the midpoint of the line segment [AB]

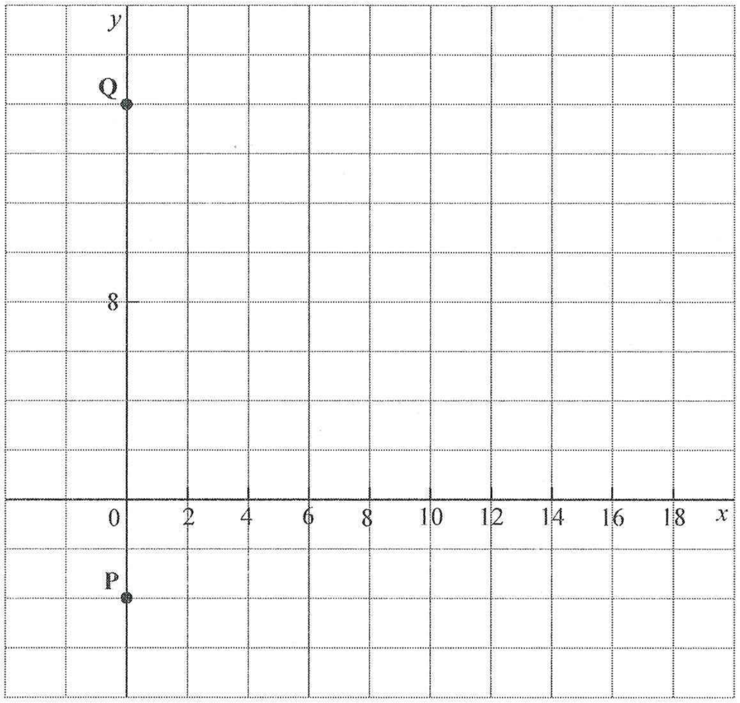
(c) Find the length of the line segment [AB]

(d) Find the gradient of a line perpendicular to the line segment [AB].

(e) The line $2x + by - 12 = 0$ is perpendicular to the line segment [AB]. What is the value of b?

(Total 8 marks)

3. **Points P(0,–4), Q (0, 16) are shown on the diagram.**



(a) Plot the point R (11,16).

(b) Calculate angle $Q\hat{P}R$.

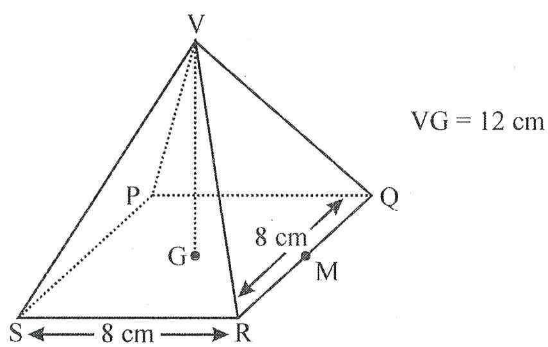
M is a point on the line PR. M is 9 units from P.

(c) Calculate the area of triangle PQM.

(Total 6 marks)

4. In the diagram below, PQRS is the square base of a solid right pyramid with vertex V. The sides of the square are 8 cm, and the height VG is 12 cm. M is the midpoint of [QR].

Diagram not to scale



(a) (i) Write down the length of [GM].

(ii) Calculate the length of [VM].

(2)

(b) Find

(i) the total surface area of the pyramid;

(ii) the angle between the face VQR and the base of the pyramid.

(4)
(Total 6 marks)

The height of a vertical cliff is 450 m. The angle of elevation from a ship to the top of the cliff is 23°. The ship is x metres from the bottom of the cliff.

(a) Draw a diagram to show this information.

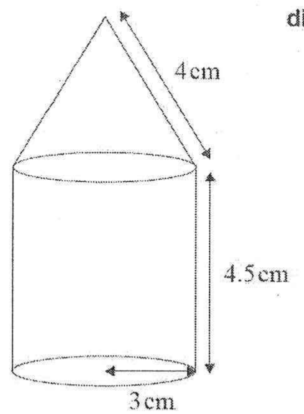
Diagram:

(b) Calculate the value of x.

(Total 4 marks)

2. [Maximum mark: 17]
The following diagram shows a perfume bottle made up of a cylinder and a cone.

diagram not to scale



The radius of both the cylinder and the base of the cone is 3 cm.
The height of the cylinder is 4.5 cm.
The slant height of the cone is 4 cm.
(a) (i) Show that the vertical height of the cone is 2.65 cm correct to three significant figures.

(ii) Calculate the volume of the perfume bottle. [6]

The bottle contains $125\ \text{cm}^3$ of perfume. The bottle is **not** full and all of the perfume is in the cylinder part.

(b) Find the height of the perfume in the bottle. [2]

Temi makes some crafts with perfume bottles, like the one above, once they are empty. Temi wants to know the surface area of one perfume bottle.

(c) Find the **total** surface area of the perfume bottle. [4]

Temi covers the perfume bottles with a paint that costs 3 South African rand (ZAR) per millilitre. One millilitre of this paint covers an area of $7\ \text{cm}^2$.

(d) Calculate the cost, in ZAR, of painting the perfume bottle. **Give your answer correct to two decimal places**. [3]

Temi sells her perfume bottles in a craft fair for 325 ZAR each. Dominique from France buys one and wants to know how much she has spent, in euros (EUR). The exchange rate is

1 EUR = 13.03 ZAR.

(e) Find the price, in EUR, that Dominique paid for the perfume bottle. **Give your answer** correct to two decimal places. **[2]**

4. [Maximum mark: 21]

A boat race takes place around a triangular course, ABC, with $\mathrm{AB} = 700\,\mathrm{m}$, $\mathrm{BC} = 900\,\mathrm{m}$ and angle $\mathrm{ABC} = 110^\circ$. The race starts and finishes at point A.

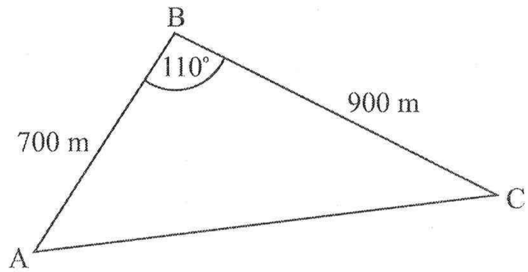


diagram not to scale

(a) Calculate the total length of the course. [4]

It is estimated that the fastest boat in the race can travel at an average speed of $1.5\,\mathrm{m\,s^{-1}}$.

(b) Calculate an estimate of the winning time of the race. Give your answer to the nearest minute. [3]

(c) Find the size of angle ACB. [3]

To comply with safety regulations, the area inside the triangular course must be kept clear of other boats, and the shortest distance from B to AC must be greater than 375 metres.

(d) Calculate the area that must be kept clear of boats. [3]

(e) Determine, giving a reason, whether the course complies with the safety regulations. [3]

The race is filmed from a helicopter, H, which is flying vertically above point A. The angle of elevation of H from B is 15°.

(f) Calculate the vertical height, AH, of the helicopter above A. [2]

(g) Calculate the maximum possible distance from the helicopter to a boat on the course. [3]

12. An iron bar is heated. Its length, L, in millimetres can be modelled by a linear function, $L = mT + c$, where T is the temperature measured in degrees Celsius ($^\circ$C).

At $150\,^\circ\text{C}$ the length of the iron bar is $180\,\text{mm}$.

(a) Write down an equation that shows this information. [1]

At $210\,^\circ\text{C}$ the length of the iron bar is $181.5\,\text{mm}$.

(b) Write down an equation that shows this second piece of information. [1]

(c) Hence, find the length of the iron bar at $40\,^\circ\text{C}$. [4]

Working:

Answers:

(a) . . . .

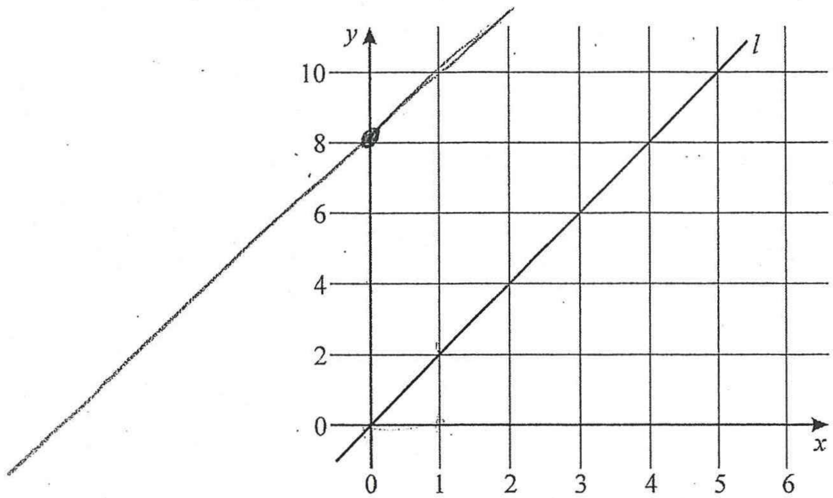
(b) . . . .

(c) . . . .

IB Math Studies SL 11 Ch13 Coordinate Geometry And Geometry Review

Name KEY

1. The following diagram shows a straight line l.



(a) Find the equation of the line l.

$y = 2x$

(b) The line n is parallel to l and passes through the point (0, 8). Write down the equation of the line n.

$y = 2x + 8$

(c) The line n crosses the horizontal axis at the point P. Find the coordinates of P. x-int

$0 = 2x + 8$ $x = -4$

(Total 4 marks)

2. **A is the point (2, 3), and B is the point (4, 9).**

(a) Find the gradient of the line segment [AB]. $m = \frac{9-3}{4-2} = \frac{6}{2} = 3$

(b) Find the midpoint of the line segment [AB] m.p $= \frac{2+4}{2}, \frac{3+9}{2} = (3, 6)$

(c) Find the length of the line segment [AB]

(d) Find the gradient of a line perpendicular to the line segment [AB]. $m = -\frac{1}{3}$ b/c $3 \times -\frac{1}{3} = -1$

(e) The line $2x + by - 12 = 0$ is perpendicular to the line segment [AB]. What is the value of b?

c) $d = \sqrt{(2-4)^2 + (3-9)^6}$

$= \sqrt{4 + 36}$

$= \sqrt{40}$

$= 6.32$

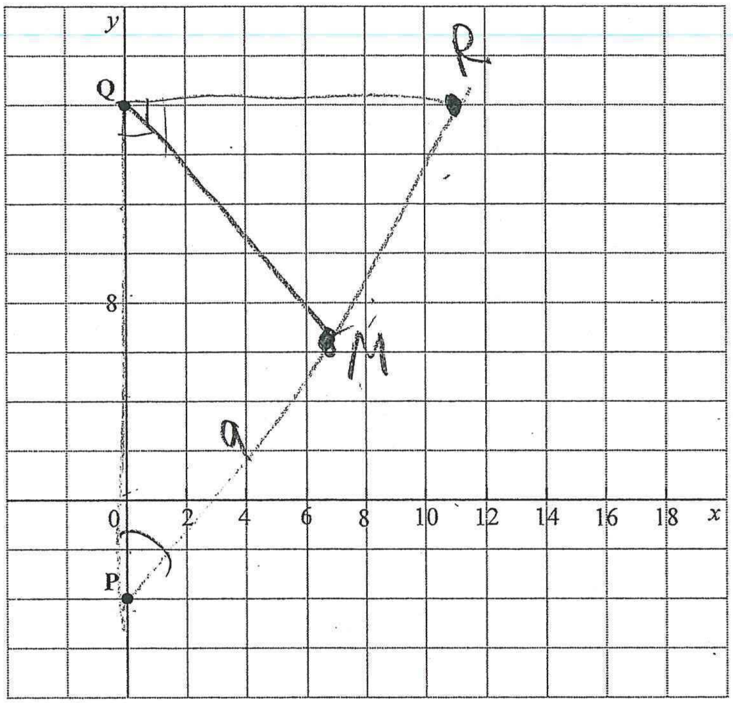
$y = \frac{-2}{6}x + \frac{12}{6}$

$b = 6$

(Total 8 marks)

$m = -\frac{A}{b}$ $\frac{-1}{3} = \frac{-2}{6}$

3. **Points P(0,–4), Q (0, 16) are shown on the diagram.**



(a) Plot the point R (11,16).

(b) Calculate angle $Q\hat{P}R$. $\tan QPR = \frac{11}{20}$ $\angle QPR \doteq 28.8°$

M is a point on the line PR. M is 9 units from P.

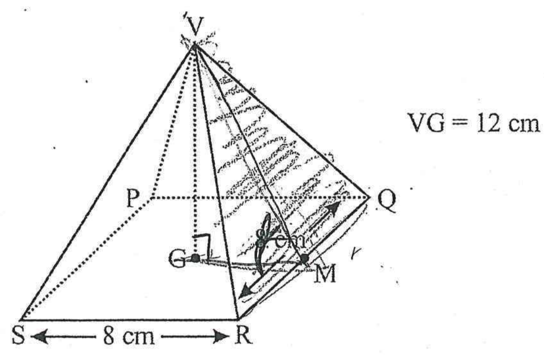
(c) Calculate the area of triangle PQM.

$A = \frac{1}{2}(20)(9)\sin 28.8$
$= 43.4$ units^2

(Total 6 marks)

4. In the diagram below, PQRS is the square base of a solid right pyramid with vertex V. The sides of the square are 8 cm, and the height VG is 12 cm. M is the midpoint of [QR].

Diagram not to scale



(a) (i) Write down the length of [GM].

4

(ii) Calculate the length of [VM]. **(2)**

$VM = \sqrt{4^2 + 12^2}$

$= 12.649 \doteq 12.6$

(b) Find

(i) the total surface area of the pyramid;

$S.A. = 4\left(\frac{1}{2}\, 8 \cdot 12.6\right) + 8^2$

$= 265.6$

(ii) the angle between the face VQR and the base of the pyramid. **(4)**

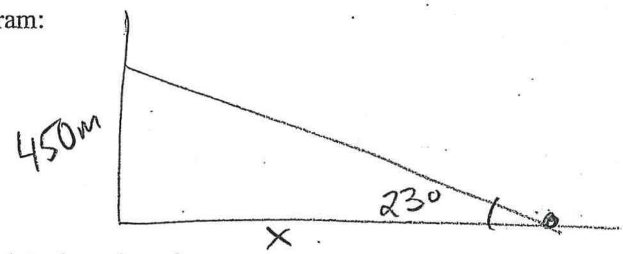
$\tan \cancel{VQR}\ \widehat{VMG} = \frac{VG}{GM} = \frac{12}{4} = 3 \quad \tan^{-1}(3) = 71.565 \doteq 71.6$

(Total 6 marks)

The height of a vertical cliff is 450 m. The angle of elevation from a ship to the top of the cliff is 23°.
The ship is x metres from the bottom of the cliff.

(a) Draw a diagram to show this information.

Diagram:



$x = 1060\text{ m}$

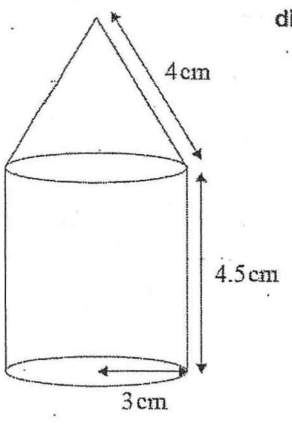
(b) Calculate the value of x.

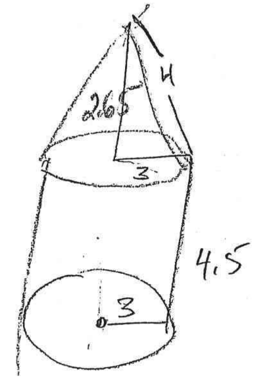
$\tan 23° = \frac{450}{x}$

(Total 4 marks)

2. [Maximum mark: 17]
The following diagram shows a perfume bottle made up of a cylinder and a cone.

diagram not to scale





The radius of both the cylinder and the base of the cone is 3 cm.
The height of the cylinder is 4.5 cm.
The slant height of the cone is 4 cm.
(a) (i) Show that the vertical height of the cone is 2.65 cm correct to three significant figures.

$\sqrt{4^2-3^2}=h \quad h=2.65$

(ii) Calculate the volume of the perfume bottle. [6]

$V=\pi r^2 h+\frac{1}{3}\pi r^2 s$
$=\pi 3^2\cdot 4.5+\frac{1}{3}\pi\cdot 3^2\cdot 4 \quad =164.933 \doteq 165\text{ mL}$

The bottle contains 125 cm^3 of perfume. The bottle is **not** full and all of the perfume is in the cylinder part.

(b) Find the height of the perfume in the bottle. [2]

$\pi 3^2\cdot h=125 \quad h=\frac{125}{\pi 3^2}$
$=4.42\text{cm}$

Temi makes some crafts with perfume bottles, like the one above, once they are empty.
Temi wants to know the surface area of one perfume bottle.

(c) Find the **total** surface area of the perfume bottle. [4]

$S.A.=\pi r l+2\pi r h$
$=\pi 3\cdot 4+2\pi(3)\times(4.5) \quad =123 \text{ cm}^2$

Temi covers the perfume bottles with a paint that costs 3 South African rand (ZAR) per millilitre. One millilitre of this paint covers an area of 7 cm^2.

(d) Calculate the cost, in ZAR, of painting the perfume bottle. **Give your answer correct to two decimal places**. [3]

$123\text{ cm}^2\times\frac{3\text{ zar}}{7\text{ cm}^2}=52.71\text{ zar}$

Temi sells her perfume bottles in a craft fair for 325 ZAR each. Dominique from France buys one and wants to know how much she has spent, in euros (EUR). The exchange rate is
1 EUR = 13.03 ZAR.

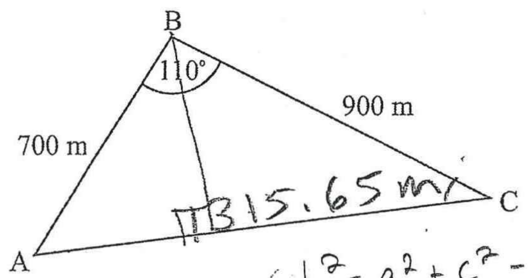
(e) Find the price, in EUR, that Dominique paid for the perfume bottle. **Give your answer** correct to two decimal places. **[2]**

$325\text{ ZAR}\times\frac{1\text{ EUR}}{13.03\text{ ZAR}}=24.94\text{ EUR}$

4. **[Maximum mark: 21]**

A boat race takes place around a triangular course, ABC, with $AB = 700\,m$, $BC = 900\,m$ and angle $ABC = 110°$. The race starts and finishes at point A.

diagram not to scale



$b^2 = a^2 + c^2 - 2ac\cos B$

$= 700^2 + 900^2 - 2(700)(900)\cos 110°$

(a) Calculate the total length of the course. [4]

$700 + 900 + 1315.65\,m = 2915.65\,m \doteq 2920\,m$

It is estimated that the fastest boat in the race can travel at an average speed of $1.5\,m\,s^{-1}$.

(b) Calculate an estimate of the winning time of the race. Give your answer to the nearest minute. [3]

$2920\,m \times \frac{s}{1.5\,m} = \frac{1947\,s}{60\,s} = 32\,min$

(c) Find the size of angle ACB. [3]

$\frac{\sin C}{700\,m} = \frac{\sin 110}{1316\,m} = 29.989 \doteq 30.0°$

To comply with safety regulations, the area inside the triangular course must be kept clear of other boats, and the shortest distance from B to AC must be greater than 375 metres.

(d) Calculate the area that must be kept clear of boats. [3]

$\frac{1}{2} \times 700 \times 900 \times \sin 110° \doteq 296000\,m^2$

(e) Determine, giving a reason, whether the course complies with the safety regulations. [3]

$\sin 30° = \frac{x}{900}$ $d \doteq 450\,m$ $450\,m > 375\,m$ so it complies

The race is filmed from a helicopter, H, which is flying vertically above point A. The angle of elevation of H from B is 15°.

(f) Calculate the vertical height, AH, of the helicopter above A. [2]

(g) Calculate the maximum possible distance from the helicopter to a boat on the course. [3]

f) $\tan 15° = \frac{AH}{700}$

$AH = 188\,m$

g) $HC^2 = 187.564^2 + 1315.65^2$

$HC = 1330\,m$

12. An iron bar is heated. Its length, L, in millimetres can be modelled by a linear function, $L = mT + c$, where T is the temperature measured in degrees Celsius (°C).

At $150\,°\text{C}$ the length of the iron bar is $180\,\text{mm}$.

(a) Write down an equation that shows this information. [1]

At $210\,°\text{C}$ the length of the iron bar is $181.5\,\text{mm}$.

(b) Write down an equation that shows this second piece of information. [1]

(c) Hence, find the length of the iron bar at $40\,°\text{C}$. [4]

Working:

a) $\overset{y_2}{180} = \overset{x_2}{150}m + c$

b) $\overset{y_1}{181.5} = \overset{x_1}{210}m + c$

$m = \dfrac{181.5 - 180}{210 - 150}$

$= \dfrac{1.5}{60}$

$= 0.025$

$180 = 150(.025) + c$

$c = 176.25$

c) $L = 40(0.025) + 176.25$

$= 177.25$

$\doteq 177\text{mm}$

Answers:

(a) ....

(b) ....

(c) ....

16EP13